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Solution of the renormalization problem in non-relativistic quantum electrodynamics

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Abstract. The renormalization problem is solved without using the mass renormalization concept, by introducing a new physical interaction picture in which the unobservable interaction of the free electron with the vacuum radiation field is eliminated. This is achieved by applying a self-consistent projection-operator method, developed recently by the present author. A finite expression for the non-relativistic Lamb shift, whose explicit evaluation shows a fairly good agreement with the experiment, is derived.

As Au and Feinberg [1] have pointed out, ambiguities exist in the mass renormalization procedure in the case of non-relativistic treatment of the Lamb shift. The aim of the present paper is to show that a new interaction picture, in which finite results without renormalization follow automatically, can be found.

The renormalization concept, introduced by Kramers [2] and firstly applied by Bethe [3] for the calculation of the non-relativistic Lamb shift, consists of replacing the experimentally unobservable 'bare' mass of a free electron by its 'renormalized', experimental mass (being the sum of the electromagnetic mass and bar, mechanical mass). The idea that only physically observable quantities should play a role in the theory is a very old one and stems from Heisenberg.

However, in our opinion a more natural, completely self-consistent and relatively simple solution of the problem can be achieved without the mass renormalization concept, by introducing a new 'physical' interaction picture, in which the physically unobservable interaction of the free electron with the vacuum radiation field is eliminated. It is self-evident that a self-consistent calculation of the Lamb (radiative-level) shift can be carried out only by studying the time behaviour of the atomic system (bound electron) interacting with the radiation field. As will be shown in this paper, such a treatment can be carried out by applying the self-consistent projection-operator method (SCPOM) which has been recently developed and applied by the present author [4–9].

The Hamiltonians for the unperturbed hydrogenic atom and the quantized radiation field read as

$$H_A = \left[\frac{P^2}{2m} + V(R)\right] \otimes I_S \qquad H_R = \hbar c \sum_{\lambda=1}^{2} \int \mathrm{d}^3 k \, k a^+(k,\lambda) a^-(k,\lambda) \tag{1}$$

where I_S is the unit operator in the spin Hilbert space, $a^{\pm}(k,\lambda)$ are the photon creation and annihilation operators for the mode (k,λ) and $e_{k,\lambda}$ is the polarization

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vector (λ is the polarization index). Further, the interaction Hamiltonian between the atom and the radiation field is given by

$$H_I = -\frac{e}{mc}\boldsymbol{A}(\boldsymbol{R})\cdot\boldsymbol{P} - \frac{e\hbar}{2mc}\sigma\cdot\boldsymbol{B}(\boldsymbol{R}) \tag{2}$$

where

$$\boldsymbol{A}(\boldsymbol{R}) = \frac{\sqrt{\hbar c}}{2\pi}\sum_{\lambda=1}^{2}\int\frac{d^3k}{\sqrt{k}}\boldsymbol{e}_{\boldsymbol{k},\lambda}\left(a^-(\boldsymbol{k},\lambda)e^{i\boldsymbol{k}\cdot\boldsymbol{R}} + a^+(\boldsymbol{k},\lambda)e^{-i\boldsymbol{k}\cdot\boldsymbol{R}}\right) \tag{3}$$

is the vector potential, σ is the Pauli spin operator and $\boldsymbol{B} = \nabla\times\boldsymbol{A}$ is the magnetic field.

Since we are interested in the calculation of the radiative energy (Lamb) shift of the state $|nlm, m_s, v\rangle = |nlm\rangle\otimes|m_s\rangle\otimes|v\rangle$ with the energy eigenvalue E_n (n, l, m, m_s are the principal, angular momentum, magnetic and spin-magnetic quantum numbers, respectively and v denotes the vacuum state of the radiation field), we choose for our projection operator $\mathcal{P}$

$$\mathcal{P} = |nlm, m_s, v\rangle\langle nlm, m_s, v| \tag{4}$$

Then, by applying the SCPOM [4–9] to the Schrödinger equation in the interaction picture

$$\frac{d|\psi(t)\rangle}{dt} = -\frac{i}{\hbar}H_I(t)|\psi(t)\rangle \qquad |\psi(0)\rangle = \mathcal{P}|\psi(0)\rangle = |nlm, m_s, v\rangle \tag{5}$$

$$H_I(t) \equiv \exp[(i/\hbar)(P^2/2m + V + H_R)t]H_I\exp[-(i/\hbar)(P^2/2m + V + H_R)t] \tag{6}$$

an exact closed equation of motion for the reduced state vector $\mathcal{P}|\psi(t)\rangle$ can be obtained [4]

$$\frac{d\mathcal{P}|\psi(t)\rangle}{dt} = -\frac{1}{\hbar^2}\int_0^t d\tau\,\mathcal{P}H_I(t)U(t, t-\tau)H_I(t-\tau)\mathcal{P}|\psi(t-\tau)\rangle \tag{7}$$

$$U(t, t-\tau) = \mathcal{T}\exp\left[-\frac{i}{\hbar}\int_{t-\tau}^{t}dt'\,(I-\mathcal{P})H_I(t')\right] \tag{8}$$

where $\mathcal{T}$ is the Dyson time-ordering operator, I is the unit operator in the product Hilbert space $\mathcal{H}_A\otimes\mathcal{H}_R$ of the atom and radiation field, and we took into account that $\mathcal{P}H_I\mathcal{P} = 0$.

A new 'physical' interaction picture will be now introduced. This new picture, in which the dynamics of the free electron interacting with the radiation field is eliminated, will be referred to as the 'bound-electron interaction picture'. To achieve this, it is necessary to describe the time evolution of a free electron, being initially in the state

$$|nlm, m_s\rangle = \int d^3p\,\langle\boldsymbol{p}|nlm, m_s\rangle|\boldsymbol{p}\rangle \tag{9}$$

(which is a superposition of free-electron eigenstates $|p\rangle$), under the influence of its interaction with the vacuum radiation field.

The application of the SCPOM to the Schrödinger equation (5), which in the case of the free electron ($V = 0$) reduces to

$$\frac{\mathrm{d}|\tilde{\psi}(t)\rangle}{\mathrm{d}t} = -\frac{\mathrm{i}}{\hbar}\tilde{H}_I(t)|\tilde{\psi}(t)\rangle \qquad |\tilde{\psi}(0)\rangle = |\psi(0)\rangle = |nlm, m_s, v\rangle \tag{10}$$

$$\tilde{H}_I(t) \equiv \exp[(\mathrm{i}/\hbar)(P^2/2m + H_R)t] H_I \exp[-(\mathrm{i}/\hbar)(P^2/2m + H_R)t] \tag{11}$$

leads to an exact closed equation of motion for $\mathcal{P}|\tilde{\psi}(t)\rangle$ describing the time evolution of the reduced state vector under the influence of the free-electron-field interaction

$$\frac{\mathrm{d}\mathcal{P}|\tilde{\psi}(t)\rangle}{\mathrm{d}t} = -\frac{1}{\hbar^2}\int_0^t \mathrm{d}\tau\, \mathcal{P}\tilde{H}_I(t)\tilde{U}(t, t-\tau)\tilde{H}_I(t-\tau)\mathcal{P}|\tilde{\psi}(t-\tau)\rangle \tag{12}$$

with $\tilde{U}(t, t-\tau)$ defined by (8) in which all Hamiltonians H_I are replaced by $\tilde{H}_I$. This equation in the so-called Markov approximation (which consists of neglecting memory effects) [4, 5] reduces to

$$\frac{\mathrm{d}\mathcal{P}|\tilde{\psi}(t)\rangle}{\mathrm{d}t} = A_{\mathrm{free}}(t, 0)\mathcal{P}|\tilde{\psi}(t)\rangle \tag{13}$$

$$A_{\mathrm{free}}(t, 0) \equiv -\frac{1}{\hbar^2}\int_0^t \mathrm{d}\tau\, \langle nlm, m_s, v|\tilde{H}_I(t)\tilde{U}(t, t-\tau)\tilde{H}_I(t-\tau)|nlm, m_s, v\rangle \tag{14}$$

whose solution reads as

$$\mathcal{P}|\tilde{\psi}(t)\rangle = \exp\left[\int_0^t \mathrm{d}\tau\, A_{\mathrm{free}}(\tau, 0)\right]|\psi(0)\rangle. \tag{15}$$

The solution (15) does not contain a time-ordering operator since $A_{\mathrm{free}}(t, 0)$ is an expectation value, and therefore the commutation relation $[A_{\mathrm{free}}(t_1, 0), A_{\mathrm{free}}(t_2, 0)] = 0$ holds for arbitrary times t_1 and t_2.

Now, we can introduce the 'bound-electron interaction picture'

$$\mathcal{P}|\psi(t)\rangle_{\mathrm{BEI}} \equiv \exp\left[-\int_0^t \mathrm{d}\tau\, A_{\mathrm{free}}(\tau, 0)\right]\mathcal{P}|\psi(t)\rangle. \tag{16}$$

In the Markov approximation, equation (7), in this new picture, takes the form

$$\frac{\mathrm{d}\,P|\psi(t)\rangle_{\mathrm{BEI}}}{\mathrm{d}t} = (-\mathrm{i}/\hbar)\Delta A(t, 0)P|\psi(t)\rangle_{\mathrm{BEI}} \tag{17}$$

$$\Delta A(t, 0) \equiv \mathrm{i}\hbar[A_{\mathrm{bound}}(t, 0) - A_{\mathrm{free}}(t, 0)] \tag{18}$$

$$A_{\mathrm{bound}}(t, 0) \equiv -\frac{1}{\hbar^2}\int_0^t \mathrm{d}\tau\, \langle nlm, m_s, v|H_I(t)U(t, t-\tau) \times H_I(t-\tau)|nlm, m_s, v\rangle \tag{19}$$

with the solution given by

$$\mathcal{P}|\psi(t)\rangle_{\mathrm{BEI}} = \exp\left[(-\mathrm{i}/\hbar)\int_0^t \mathrm{d}\tau\,\Delta A(\tau,0)\right]|\psi(0)\rangle. \tag{20}$$

In this new picture, a complete elimination of the dynamics of the free electron interacting with the vacuum radiation field is performed to all orders of interaction H_I. This procedure is unambiguous and self-consistent. It is easy to see that in the second-order approximation as to the strength of the interaction (Born approximation), the expression $\Delta A(t\to\infty,0)$ (where the upper limit of the time integration is replaced with ∞ by introducing a damping factor $\mathrm{e}^{-e\tau}$ in the integral ($e\to 0$ after carrying out the integration)) reduces to a 'renormalized' expression for the radiative energy-level shift

$$\Delta A^{(2)}(t\to\infty,0) = \Delta E^{\mathrm{bound}}_{nlm} - \Delta E^{\mathrm{free}}_{nlm} \tag{21}$$

where $\Delta E^{\mathrm{bound}}_{nlm}$ (subscript m_s is dropped) is the self-energy of the electron bound in the atom

$$\begin{aligned}\Delta E^{\mathrm{bound}}_{nlm} &= \mathrm{i}\hbar A^{(2)}_{\mathrm{bound}}(t\to\infty,0)\\ &= \frac{-\mathrm{ie}^2}{(2\pi)^2 m^2 \mathrm{c}}\sum_{\lambda}\lim_{e\to 0}\int_0^\infty \mathrm{d}\tau\,\mathrm{e}^{-e\tau}\int\frac{\mathrm{d}^3k}{k}\\ &\quad\times\langle nlm,m_s|h_I\exp\{(\mathrm{i}/\hbar)[E_n-(\boldsymbol{P}-\boldsymbol{K})^2/2m-V-cK]\tau\}h_I|nlm,m_s\rangle\end{aligned} \tag{22}$$

$$\boldsymbol{K}\equiv\hbar\boldsymbol{k}\qquad h_I = h_1+h_2\qquad h_1\equiv e_{\boldsymbol{k},\lambda}\cdot\boldsymbol{P}\qquad h_2\equiv\frac{\sigma}{2}\cdot(\hbar\boldsymbol{k}\times e_{\boldsymbol{k},\lambda}) \tag{23}$$

and $\Delta E^{\mathrm{free}}_{nlm}$ is the self-energy of the free electron

$$\begin{aligned}\Delta E^{\mathrm{free}}_{nlm} &= \mathrm{i}\hbar A^{(2)}_{\mathrm{free}}(t\to\infty,0)\\ &= \frac{-\mathrm{ie}^2}{(2\pi)^2 m^2 \mathrm{c}}\sum_{\lambda}\lim_{e\to 0}\int_0^\infty \mathrm{d}\tau\,\mathrm{e}^{-e\tau}\int\frac{\mathrm{d}^3k}{k}\\ &\quad\times\langle nlm,m_s|h_I\exp[(\mathrm{i}/\hbar)(-\boldsymbol{K}^2/2m+\boldsymbol{P}\cdot\boldsymbol{K}/m-\mathrm{c}K)\tau]h_I|nlm,m_s\rangle.\end{aligned} \tag{24}$$

In obtaining the above equations we inserted the Hamiltonians from (1) and (2), and used the relations: $\langle v|a^-(\boldsymbol{k},\lambda)a^+(\boldsymbol{k}',\lambda')|v\rangle = \mathrm{d}(\boldsymbol{k}-\boldsymbol{k}')\mathrm{d}_{\lambda,\lambda'}$ and

$$\begin{aligned}&\mathrm{e}^{(\mathrm{i}/\hbar)H_R t}a^{\pm}(\boldsymbol{k},\lambda)\mathrm{e}^{-(\mathrm{i}/\hbar)H_R t} = \mathrm{e}^{\pm\mathrm{c}Kt}a^{\pm}\\ &\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{R}}\mathrm{e}^{(\mathrm{i}/\hbar)(\boldsymbol{P}^2/2m)t}\mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{R}} = \mathrm{e}^{(\mathrm{i}/\hbar)[(\boldsymbol{P}-\boldsymbol{K})^2/2m]t}\end{aligned} \tag{25}$$

After carrying out the integration over τ, the expression for the Lamb shift takes the following form:

$$\Delta E_{nlm} = \Delta E_{nlm}(h_1) + \Delta E_{nlm}(h_2) \tag{26}$$

$$\Delta E_{nlm}(h_e) = \frac{e^2\hbar}{(2\pi)^2 m^2 c} \sum_{\lambda} \int \frac{d^3k}{k} \langle nlm, m_s | h_e (G_{\text{bound}} - G_{\text{free}}) h_e | nlm, m_s \rangle$$

$$e = 1, 2 \tag{27}$$

where the Green operators G_{bound} and G_{free} are given by

$$G_{\text{bound}} = \frac{1}{E_n - (\boldsymbol{P} - \boldsymbol{K})^2/2m - V(R) - cK}$$

$$G_{\text{free}} = \frac{-1}{K^2/2m - \boldsymbol{K} \cdot \boldsymbol{P}/m + cK} \tag{28}$$

In this way, without using any renormalization we obtained a finite expression for the Lamb shift.

In order to get explicitly calculable expressions, an expansion in powers of $V(R)$ should be carried out

$$\Delta E_{nlm}(h_e) = \Delta E_{nlm,1}(h_e) + \Delta E_{nlm,2}(h_e) + \Delta E_{nlm,3}(h_e) \qquad e = 1, 2 \tag{29}$$

with

$$\Delta E_{nlm,i}(h_e) = \frac{e^2\hbar c}{m^2 c^2 (2\pi)^2} \sum_{\lambda} \int \frac{d^3k}{k} \langle nlm, m_s | h_e G_i h_e | nlm, m_s \rangle$$

$$i = 1, 2, 3 \tag{30}$$

$$G_1 \equiv G_0 - G_{\text{free}} \qquad G_2 \equiv G_0 V G_0 \qquad G_3 \equiv G_0 V G_0 V G_{\text{bound}} \tag{31}$$

where the Green operator G_0 containing the complete self-energy contribution of the free electron reads as

$$G_0 \equiv \frac{1}{E_n - (\boldsymbol{P} - \boldsymbol{K})^2/2m - cK}. \tag{32}$$

Now, in order to demonstrate the finiteness of our results, explicit calculations of the Lamb shift will be carried out by taking into account retardation effects. This will be achieved, quite analogously to Lieber [10], but with the significant difference that retardation effects will be taken into account, by using the O(4) symmetry of the non-relativistic hydrogen atom [11]. In this way, the introduction of a relativistic cutoff [3, 10], being inconsistent with the simultaneously used dipole approximation (which requires a much lower cutoff frequency), can be avoided. In the following we will expand $\Delta E_{nlm,i}(h_e)$ in powers of $\boldsymbol{P} \cdot \boldsymbol{K}$ and take only contributions up to second order into account (since higher order contributions are neglegibly small). The zero-order $\boldsymbol{P} \cdot \boldsymbol{K}$ contributions read as [12]

$$\Delta E^{(0)}_{nlm,1}(h_1) = -\frac{2\alpha^3}{3\pi m} \int dK K (1-\rho)^2 \left(L(\rho, nlm) - \frac{1}{2\rho} \right) \tag{33}$$

$$\Delta E^{(0)}_{nlm,2}(h_1) = \frac{-4\alpha n}{3\pi m} \int K dK (1-\rho)^4 L'(\rho, nlm) \tag{34}$$

where L and L', apart from a different ρ (which contains retardation)

$$\frac{2\rho}{(1-\rho)^2} \equiv \frac{1}{2p_n^2}(K^2 + 2mcK) \qquad p_n \equiv \sqrt{-2mE_n} \qquad E_n \equiv -\frac{\alpha^2 mc^2}{2n^2}$$

are analogously defined as in [10] (α is the fine-structure constant). For 1s, 2s and 2p states L and L' take the form [12]

$$L(\rho,100) = \frac{3-\rho^2}{4} \qquad L(\rho,200) = \frac{(1+\rho^2)(2-\rho^2)}{4}$$

$$L(\rho,21m) = \frac{10-5\rho^2+\rho^4}{12} \qquad L'(\rho,100) = \frac{1}{8\rho^2} + \frac{(1-\rho^2)^2}{8\rho^4}\ln|1-\rho^2|$$

$$L'(\rho,200) = \frac{1}{4}\left[\frac{1}{2} + \rho^2 - \frac{\rho^4}{2} - \frac{\rho^6}{4} - \frac{(1-\rho^4)^2}{2}\sum_{n=3}^{\infty}\frac{\rho^{2n-6}}{n}\right]$$

$$L'(\rho,21m) = \frac{1}{24\rho^2} \times \left\{(1+\rho^2)(3-\rho^2) - \frac{3}{2} - \frac{(1-\rho^2)^2}{\rho^2}\left[1 + \frac{(1-\rho^2)^2}{\rho^2}\ln(1-\rho^2)\right]\right\}.$$

This latter leads to the following numerical results

$$\begin{aligned} \Delta E^{(0)}_{100,1}(h_1)/h &= 12029.7 \text{ MHz} \\ \Delta E^{(0)}_{200,1}(h_1)/h &= 1841.5 \text{ MHz} \\ \Delta E^{(0)}_{21m,1}(h_1)/h &= 573.3 \text{ MHz} \end{aligned} \tag{35}$$

$$\begin{aligned} \Delta E^{(0)}_{100,2}(h_1)/h &= -3963.5 \text{ MHz} \\ \Delta E^{(0)}_{200,2}(h_1)/h &= -790.234 \text{ MHz} \\ \Delta E^{(0)}_{21m,2}(h_1)/h &= -521.933 \text{ MHz} \end{aligned} \tag{36}$$

(where h is the Planck constant).

The third term $\Delta E^{(0)}_{nlm,3}(h_1)$, which includes all remaining orders in the potential V, can similarly be calculated as in [10]

$$\Delta E^{(0)}_{100,3}(h_1)/h = -\frac{\alpha^3 mc^2}{3\pi}\int_0^1 \mathrm{d}\rho\,\frac{(1-\rho)(1-\rho^2)^3[w_1 - 1 + \rho]}{w_1} \times \sum_{m=2}^{\infty}\frac{m-1}{m}\frac{\rho^{2m-4}}{\rho + (m-1)/(m+1)}$$

$$= -130.21 \text{ MHz} \tag{37}$$

$$\Delta E^{(0)}_{200,3}(h_1)/h = -\frac{\alpha^3 mc^2}{12\pi h}\int_0^1 \mathrm{d}\rho\,\frac{(1-\rho)(1-\rho^2)^3[w_2-1+\rho]}{w_2} \times \sum_{m=2}^{\infty}\frac{m^2-1}{m}\frac{\rho^{2m-6}[(m+2)\rho^2-m+2]}{(m+2)\rho+m-2} = -35.93\ \text{MHz} \tag{38}$$

$$\Delta E^{(0)}_{21m,3}(h_1)/h = -\frac{\alpha^3 mc^2}{108\pi h}\mathrm{PV}\int_0^1 \mathrm{d}\rho\,\frac{(1-\rho)(1-\rho^2)^3[w_2-1+\rho]}{w_2} \times \left\{\sum_{k=1}^{\infty}\frac{\rho^{2k-6}(k+1)^2(k+2)[(k-1)(k-2)/(k+1)(k+2)-\rho^2]^2}{k[\rho+(k-2)/(k+2)]} +2(1-\rho^2)^2\sum_{k=3}^{\infty}\frac{\rho^{2k-6}(k^2-1)(k-2)}{k[(\rho+(k-2)(k+2)]}\right\} = -55.51\ \text{MHz} \tag{39}$$

where PV is the Cauchy principal value and $w_n \equiv \sqrt{(1-\rho)^2+4\alpha^2\rho/n^2}$.

Since $G^{(0)}_{\text{bound}}|nlm\rangle = G^{(0)}_{\text{free}}|nlm\rangle$, no zero-order contribution in $\boldsymbol{P}\cdot\boldsymbol{K}$ arises from the spin-radiation interaction: $\Delta E^{(0)}_{nlm}(h_2) = 0$. Furthermore, as a consequence of the angular $\vee_K$-integration ($\mathrm{d}^3K = K^2\mathrm{d}K\mathrm{d}\vee_K$), the first-order corrections in $\boldsymbol{P}\cdot\boldsymbol{K}$ vanish, $\Delta E^{(1)}_{nlm}(h_e) = 0, e = 1,2$. Therefore, the second-order corrections in $\boldsymbol{P}\cdot\boldsymbol{K}$, which read as

$$\Delta E^{(2)}_{nlm}(h_e) = \frac{3\alpha}{4\pi^2 m^2}\sum_{\lambda}\int\frac{\mathrm{d}^3K}{K}\left\langle nlm\,\middle|\, h_e\left[G^{(0)}_{\text{bound}}\left(\frac{\boldsymbol{p}\cdot\boldsymbol{K}}{m}\right)G^{(0)}_{\text{bound}} \times\left(\frac{\boldsymbol{p}\cdot\boldsymbol{K}}{m}\right)G^{(0)}_{\text{bound}} - G^{(2)}_{\text{free}}\right]h_e\,\middle|\, nlm\right\rangle \tag{40}$$

have to be evaluated. A similar calculation as above yields following numerical results

$$\Delta E^{(2)}_{100}(h_1)/h = 22.33\ \text{MHz} \qquad \Delta E^{(2)}_{200}(h_1)/h = 2.954\ \text{MHz} \tag{41}$$

$$\Delta E^{(2)}_{100}(h_2)/h = 178.6\ \text{MHz} \qquad \Delta E^{(2)}_{200}(h_2)/h = 22.32\ \text{MHz} \tag{42}$$

$$\Delta E^{(2)}_{21m}(h_e)/h \approx 0 \qquad e = 1,2 \tag{43}$$

Finally, collecting the above numerical results [cf (35)–(39) and (41)–(43)] we obtain for the Lamb shifts

$$\begin{aligned}\Delta E_{100}/h &= 8136.9\ \text{MHz}\\ \Delta E_{200}/h &= 1040.62\ \text{MHz}\\ \Delta E_{21m}/h &= -4.14\ \text{MHz}\end{aligned} \tag{44}$$

This leads to the frequency shifts $\Delta\nu(2s \to 1s) = 7096.3$ MHz and $\Delta\nu(2s \to 2p) = 1044.8$ MHz, which agrees fairly well with the corresponding experimental results [13]: 7128.9 MHz and 1057.85 MHz.

Despite the fact that relativistic Lamb shift calculations (see references quoted in [13]) lead to a better agreement with experiment, our result shows, for the first time to our knowledge, that almost the whole Lamb-shift contribution can be calculated by using a self-consistent non-relativistic treatment in which the retardation effects are taken into account. By taking into account the retardation, not only can the introduction of an inconsistent cutoff frequency (usually used in the literature [3, 10]) be avoided, but also the spin–radiation-field effect (whose contribution has been ignored in [1] and could not be taken into account in [3, 10] because of the neglect of the retardation) can be included in the Lamb shift calculation.

In spite of the apparent significance of the results obtained, the main achievement of the present paper that by using the SCPOM a new physical 'bound-electron interaction picture', in which the physically unobservable self-energy of the free electron is eliminated, can be introduced. Thus, a finite analytic expression for the non-relativistic Lamb shift can be derived without using the mass renormalization concept. In this way, for the first time to our knowledge, a self-consistent solution of the renormalization problem is presented.

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